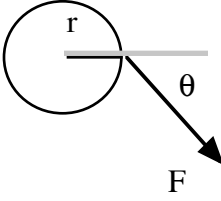


## Angular Acceleration due to Constant Torque

When a torque acts on an object, it will change its angular velocity. That is, it will experience an angular acceleration. The torque arises from a force exerted at a distance and at an angle that causes the object to spin faster or slower. The relationship between torque and angular acceleration is


$$\begin{aligned}\tau &= I\alpha \\ &= rF \sin \theta\end{aligned}$$

Here  $I$  is the moment of inertia and  $\alpha$  is the angular acceleration. Also  $r$  is the radius at which the force  $F$  is applied.

We can determine the moment of inertia experimentally if we measure the angular acceleration for different known Torques.

If the torque is constant in time, it will experience a constant angular acceleration. In this case, the angular position and velocity are given by

$$\begin{aligned}\theta_f &= \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 \\ \omega_f &= \omega_i + \alpha t\end{aligned}$$

In this experiment, we will investigate rotation under the application of a constant torque. We hope to verify that the position and velocity behave as expected. This study will allow us to determine the angular acceleration that an object experiences. Given the angular accelerations for a number of known torques, we will also determine the moment of inertia of the object.

### Procedure

To measure the angular acceleration, we need to record angular position at regular time intervals. To do this in a small lab setting, we will record linear positions over very small time intervals and convert them to angular position. We can do this since we know that

$$x = r \theta \tag{1}$$

By measuring the  $d$ 's, we can determine the  $\theta$  in radians. We will find the angular velocity in a similar way. After determining the linear velocity of the edge of our object, we can convert to the angular velocity using

$$v = r \omega \tag{2}$$

In this experiment, we will apply a constant torque to a large metal disk. We will do this by

attaching a piece of spark tape to the disk and allowing a falling mass to spin the disk. The torque applied can be related to the applied force using

$$\tau = r F \quad (3)$$

The  $F$  in this case is the tension in the tape. After the mass is released, it will fall between two charged points and a spark will make a burn mark on the tape showing position as a function of time. The sparks occur every  $1/10$  of a second.

### Detailed Procedure-Data Acquisition

1. Check to see that the apparatus is leveled properly. It is important that the mass fall perfectly vertically between the points.
2. Check to see that a clean piece of spark tape is in place for your data run. Place a your assigned mass on the tape.
3. When the sparker is engaged, the points on the apparatus are are charged with high voltage. **If you touch the apparatus while the sparker is engaged, you will be shocked. Please be careful.** While the current present is not enough to seriously harm you, you will certainly be startled.

**Make sure that everyone is clear of the apparatus before you engage the spark source.** After you are sure that you and your partners are all clear of the apparatus, you may engage the sparker. The mass should fall and leave a trail of sparks.

### Detailed Procedure-Data Analysis

1. Tape the spark record down on the lab bench so that it is flat and straight. Examine the tape and check that you can see at least 10 spark marks. They should get farther apart as you move down the tape. Pick a spark near the top of the tape and circle and number each spark mark.
2. Align a two-meter stick with the line of sparks and record the position  $x_i$  in cm of each spark. Spark one will be at 0 s, spark 2 at  $(1/10)$ s, and so on. Be sure to set the two-meter stick on end to do this.
3. Compute and record the average velocity for each interval of motion

$$v = \frac{x_{i+1} - x_i}{\Delta t}$$

where  $x$  are the positions as marked by the sparks.  $\Delta t = 1/10$  s.

4. Convert the position to angular position ( $\theta$ ) using equation (1) and plot angular position points as a function of time by hand on a piece of graph paper. Do NOT connect them together--only plot data points. What kind of curve does this appear to be?

5. Convert the velocity to angular velocity using equation (2) and plot angular velocity vs. time by hand on a piece of graph paper. Draw the best fit straight line and find the slope and intercept. The slope is the angular acceleration  $\alpha$  and the intercept is the initial angular velocity  $\omega_0$ .

6. Using the values that you have found for  $\alpha$  and  $\omega_0$ , plot the equation

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

directly on the graph that you made of  $\theta$ . How well does this graph match your experimental data?

7. Compute your applied torque ( $\tau = m(g - a)r \sin 90^\circ = mr(g - a)$ ) and angular acceleration on the board.

8. Plot  $\tau$  vs.  $\alpha$  for the class data and draw the best fitting straight line. The moment of inertia is the slope of this line.

### Questions

1. What value for the moment of inertia did you expect? How well did your value compare?
2. What assumptions have we made in doing this experiment? How do these assumptions affect your results.
3. Explain in some detail why we need such a short time interval to make our measurement?
4. Why is your  $v_0$  not equal to 0? How did you determine it from your data? Didn't we drop the mass from rest?
5. How could this experiment be improved?