Chapter 1 Problems

1.1 The Earth is approximately a sphere of radius 6.37 x 10^6 m. (a) What is is its circumference in kilometers? (b) What is its surface area in square kilometers? (c) What is its volume in cubic kilometers?

To do all three sections of this problem, we can first convert the radius to kilometers.

$$r = 6.37\times10^{6}m\cdot\frac{1km}{1000m} = 6.37\times10^{3}km$$

(a) The formula for circumference can be found in most calculus books (and in Appendix E of your Physics text). We will assume that we are finding the circumference of the equator.

$$c = 2\pi r = 2\pi\cdot 6.37\times10^{3}km = 4.002\times10^{4}km$$

(b) The surface area of a sphere is:

$$a = 4\pi r^{2} = 4\pi\cdot(6.37\times10^{3}km)^{2} = 5.099\times10^{8}km^{2}$$

(c) The volume of a sphere is:

$$v = \frac{4}{3}\pi r^{3} = \frac{4}{3}\pi\cdot(6.37\times10^{3}km)^{3} = 1.083\times10^{12}km^{3}$$

1.3 The micrometer is often called the micron. (a) How man microns make up 1 km? (b) What fraction of a centimeter equals $1\mu m$? (c) How many microns are in 1.0 yard

We begin by calculating 1 km in microns

$$d = 1.0km\cdot\frac{1000m}{1km}\cdot\frac{1micron}{1\times10^{-6}m} = 1\times10^{9}microns$$

Now calculate a distance of 1 micron in cm.

$$d = 1micron\cdot\frac{1\times10^{-6}m}{1micron}\cdot\frac{100cm}{1m} = 1\times10^{-4}cm$$

Finally we compute a distance of 1 yard in microns.

$$d = 1yd \cdot \frac{3ft}{1yd} \cdot \frac{0.3048m}{1ft} \cdot \frac{1micron}{1 \times 10^{-6}m} = 9.144 \times 10^{5} microns$$

1.7 Hydraulic engineers often use, as a unit of volume of water, the acre-foot, defined a the volume of water that will cover 1 acre of land to a depth of 1 ft. A severe thunderstorm dumps 2.0 inches of rain in 30 min. on a town of area 26 km^2. What volume of water in acre-feet, fell on the town?

We first convert the depth to feet and the area to acres.

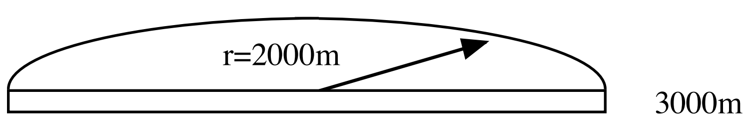
$$d = 2.0in \cdot \frac{1ft}{12in} = 0.1667ft$$

$$a = 26km^2 \cdot \left(\frac{1000m}{1km}\right)^2 \cdot \frac{2.471acres}{10^4 m^2} = 6.4245 \times 10^3 acres$$

$$Volume = a \cdot d = 6.4245 \times 10^3 acres \cdot 0.1667ft$$

$$= 1.071 \times 10^3 acre\ ft$$

1.9 Antarctica is roughly semicircular, with a radius of 2000 km. The average thickness of its ice cover is 3000m. How many cubic centimeters of ice does Antarctica contain? (Ignore the curvature of the earth)



We find the volume in cubic meters first and then convert.

$$Volume = (Area\ of\ \ half\ \ circle) \cdot (depth)$$

$$= \frac{\pi r^2}{2} \cdot d$$

$$= \frac{\pi (2000 \times 10^3 m)^2}{2} \cdot 3000m = 1.885 \times 10^{16} m^3$$

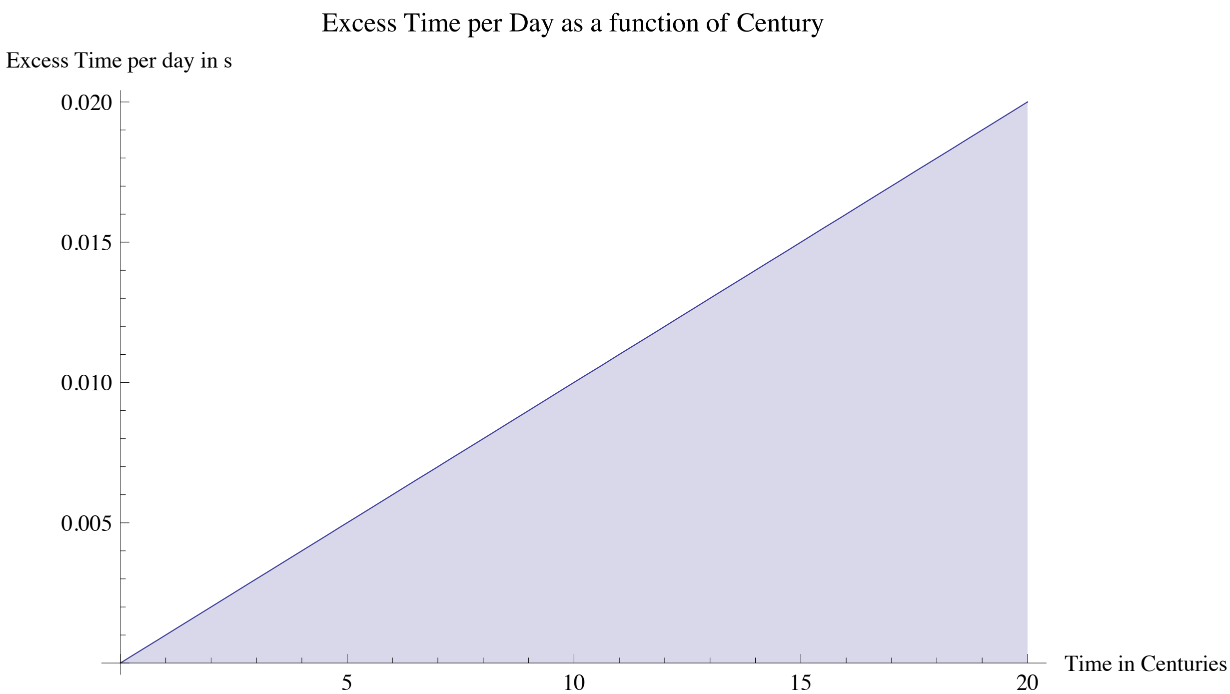
$$= 1.885 \times 10^{16} m^3 \cdot \left(\frac{100cm}{1m}\right)^3$$

$$= 1.885 \times 10^{22} cm^3$$

1.12 The fastest growing plan on record is *Hesperoyucca whipplei* that grew 3.7 m in 14 days. What was its growth rate in micrometers/second?

$$r = \frac{3.7m}{14\,days} \cdot \frac{1 micron}{1.0 \times 10^{-6} m} \cdot \frac{1\,day}{24\,hrs} \cdot \frac{1\,hr}{3600\,s} = 3.059 \frac{microns}{s}$$

1.18 Because Earth's rotation is gradually slowing, the length of each day increases: The day at the end of the 1.0 century is 1.0 ms longer than the day at the start of the century. In 20 centuries, what is the total of the daily increases in time (that is, the sum of the gain on the first day, the gain on the second day)



We did this one in class. We can plot the excess time each day as the centuries pass. The accumulated extra time is the area under the curve. We can compute this by computing the area of the triangle, remembering that the base length needs to be expressed in days, since the vertical is excess time per day.

$$t = \frac{1}{2} \cdot (20\,centuries \cdot \frac{100yr}{1century} \cdot \frac{365.25\,days}{yr}) \cdot \frac{0.020s}{day} = 7305s$$

1.21 Earth has a mass of $5.98 \times 10^{24}\,kg$. The average mass of the atoms that make up the earth is 40 u. How many atoms are there in the earth.

To proceed, we need to find the conversion of atomic mass units (u) to kilograms.

$$1\ u = 1.661 \times 10^{-27}\,kg$$

We can now proceed

$$m_{Earth} = \#\ atoms \cdot avg.\,mass\ of\ each\ atom$$

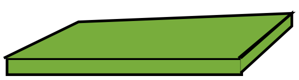
$$\#\ atoms = \frac{m_{Earth}}{avg.\,mass\ of\ each\ atom} = \frac{5.98 \times 10^{24}\,kg}{40u \cdot \frac{1.661 \times 10^{-27}\,kg}{1u}} = 9 \times 10^{49}$$

1.22. Gold, which as a mass of 19.32 g for each cubic centimeter of volume, is the most ductile metal and can be pressed into a thin leaf or drawn out into a long fiber. (a) If 1.000 oz of gold, with a mass of 27.63 g is pressed into a leaf of 1.000 microns thickness, what is the area of the leaf? (b) If, instead, the gold is drawn out into a cylindrical fiber of radius of 2.5 microns, what is the length of the fiber.

In this problem, the volume of the gold remains constant. It simply gets "reshaped" into different shapes. We compute the volume first...

$$V = 27.63g \cdot \frac{1cm^3}{19.32\,g} = 1.43\,cm^3$$

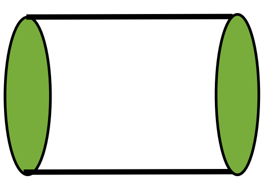
(a) In the case of a thin sheet of leaf, the volume is



$$V = Area \cdot thickness$$

$$Area = \frac{V}{thickness} = \frac{1.43cm^3}{1 \times 10^{-4}\,cm} = 1.43 \times 10^{4}\,cm^2$$

(b) In the case of a cylinder, the volume is



$$V = \pi\, r^2\, l$$

$$l = \frac{V}{\pi\, r^2} = \frac{1.43\,cm^3}{\pi\,(2.5 \times 10^{-4}\,cm)^2} = 7.28 \times 10^{6} cm$$

1.24 Grains of fine California beach sand are approximately spheres with an average radius of 50µm; They are made of silicon dioxide, 1 m^3 of which has a mass of 2600 kg. What mass of sand grains would have a total surface area equal to the surface area of a cube 1m on edge?

We begin by computing the surface area of one grain and the total area we are looking for (which is the surface area of the cube). We also simple record the density of silicon.

$$a_{sand} = 4\pi r^2 = 4\pi \cdot (50 \times 10^{-6} m)^2 = 3.14 \times 10^{-8} m^2$$
$$a_{cube} = 6l^2 = 6 \cdot (1m)^2 = 6m^2$$
$$\rho(density) = \frac{2600kg}{1m^3} = 2600\frac{kg}{m^3}$$

Now we compute the number of grains of sand. The total area should be the the number of grains times the area per grain.

$$a_{cube} = (\#\, grains) \cdot a_{sand}$$
$$\#\, grains = \frac{a_{cube}}{a_{sand}} = \frac{6m^2}{3.14 \times 10^{-8} m^2} = 1.91 \times 10^8$$

Knowing the number of grains, we can use the density to compute the mass.

$$m = (\#\, grains \cdot Volume_{grain}) \cdot \rho$$
$$= 1.91 \times 10^8 \cdot \frac{4}{3}\pi (50 \times 10^{-6} m)^3 \cdot 2600\frac{kg}{m^3}$$
$$= 2.6 \times 10^{-1} kg$$

1.26 One cubic centimeter of a typical cumulus cloud contains 50 to 500 water drops, which have a typical radius of 10 microns. For that range, give the lower value and the higher values, respectively, for the following. (as) How many cubic meters of water are in a cylindrical cumulus cloud of height 3.0 km and radius 1.0 km. (b) How many 1-liter pop bottles would that water fill? (c) Water has a mass of 1000 kg per cubic meter of volume. How much mass does the water in the cloud have?

We will calculate the lower number first. The higher values will be 10 times larger since the number of drops is 10 times higher.

a) We are told that the cloud is a cylinder. We begin by calculating the volume of the cloud in cubic cm. This will require us to convert the cloud dimensions to cm.

$$r_{cloud} = 1\,km \cdot \frac{1000m}{1\,km} \cdot \frac{100\,cm}{1m} = 1\times 10^5\,cm$$

$$h_{cloud} = 3\,km \cdot \frac{1000\,m}{1\,km} \cdot \frac{100\,cm}{1m} = 3\times 10^5\,cm$$

$$V_{cloud} = \pi r^2 h = \pi \cdot (1\times 10^5\,cm)^2 \cdot 3\times 10^5\,cm = 9.42\times 10^{15}\,cm^3$$

$$V_{water\ in\ cloud} = V_{cloud} \cdot \frac{\#\,drops}{cm^3\ of\ cloud} \cdot V_{drop}$$

$$V_{water\ in\ cloud} = 9.42\times 10^{15}\,cm^3 \cdot \frac{50 drops}{cm^3} \cdot \frac{4}{3}\pi (10\times 10^{-6}\,m)^3$$
$$= 1972.9\,m^3$$

The upper limit would be $19729\,m^3$

b. We can find the number of 1 L pop-bottles by converting the volume of water to L. The number of L will be the number of bottles.

$$1\,m^3 = 1000\,L$$

$$V_{water\ in\ cloud} = 1972.9 m^3 \cdot \frac{1000\,L}{1m^3} = 1.9729\times 10^6\,L$$

The upper limit is $1.9729\times 10^7 L$.

c We can now compute the mass of water in the cloud.

$$m = V_{water\ in cloud} \cdot \rho_{water} = 1972.9m^3 \cdot \frac{1000kg}{1m^3} = 1.9729\times 10^6\,kg$$

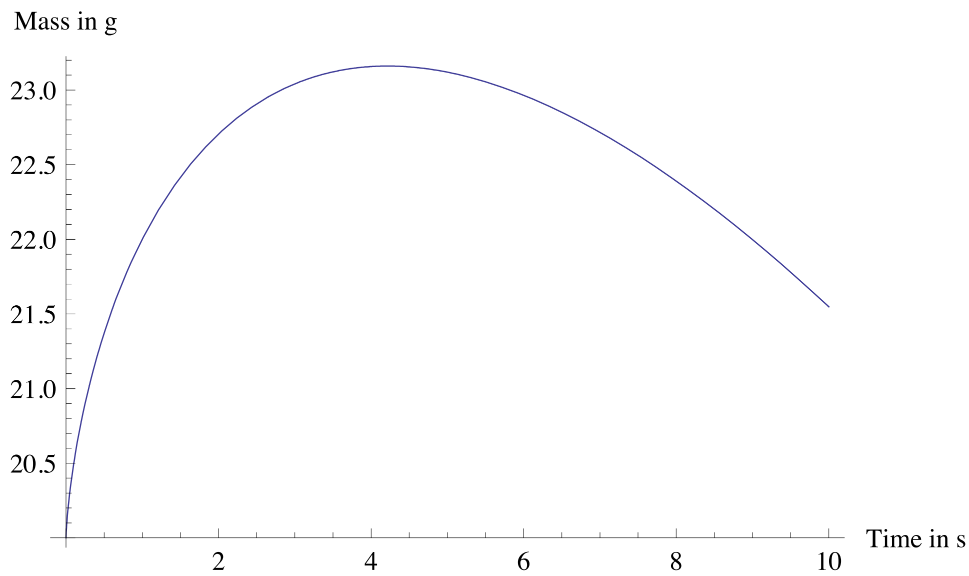
The upper limit is $1.9729\times 10^7 kg$.

1.30 Water is poured into a container that has a small leak. The mass m of the water is given as a function of time t by

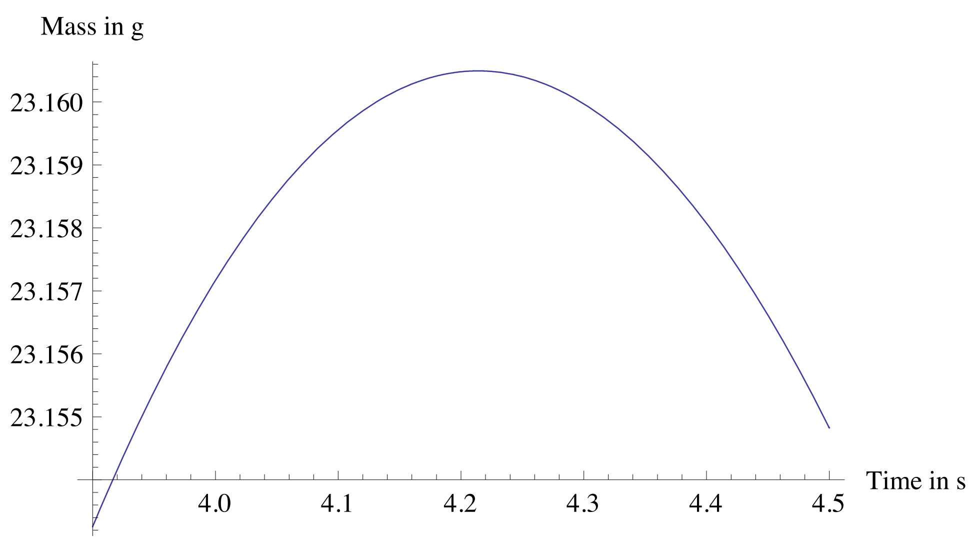
$$m = 5.00t^{0.8} - 3.00t + 20$$

with $t \ge 0$, m in grams and t in seconds. (a) At what time is the water mass greatest, and (b) what is that greatest mass. In kilograms per minute, what is the rate of mass change at (c) t=2.00 s and (d) t=5.00 s?

We could do parts (a) and (b) by plotting. First we plot the mass as a function of time for the range t=0 to t=10.



Now that we see that the peak is around 4 s, we can isolate and plot just 3.9 to 4.5.



We can do this as many times as we wish to get whatever precision we desire. The peak is at approximately 4.22 s and the mass is approximately 23.165 g.

We can also find the maximum using calculus. The maximum occurs when the slope of a curve is zero. Since the slope of a curve is given by the derivative, we could

- Find the derivative of m(t) to find the slope
- Set the derivative to 0 and solve for the time
- Find m(t) for that time

Let's try it.

- Find the derivative of m(t) to find the slope

$$\begin{aligned}\frac{dm}{dt} &= \frac{d}{dt}(5.00t^{0.8} - 3.00t + 20)\\ &= 0.8 \cdot 5.00t^{0.8-1} - 1 \cdot 3.00t^{1-1}\\ &= 4.00t^{-0.2} - 3.00\end{aligned}$$

- Set the derivative to 0 and solve for the time

$$\begin{aligned}\frac{dm}{dt} &= 4.00t^{-0.2} - 3.00\\ 0 &= 4.00t^{-0.2} - 3.00\\ 4.00t^{-0.2} &= 3.00\\ t^{-0.2} &= \frac{3.00}{4.00}\\ t &= \left(\frac{3.00}{4.00}\right)^{-5}\\ &= 4.21s\end{aligned}$$

- Find m(t) for that time

$$m = 5.00 \cdot (4.21)^{0.8} - 3.00 \cdot 4.21 + 20 = 23.16g$$

So our calculus based result matches our graphical result.

To find the rate of change, again we could approach it graphically by finding the slope of the curve using rise over run near t=2 s and t=5 s.

$$\begin{aligned}R(2) &= \frac{m(2.01) - m(1.99)}{2.01 - 1.99} = \frac{0.482g}{s}\\ &= \frac{0.482g}{s} \cdot \frac{1kg}{1000g} \cdot \frac{60s}{1\min} = \frac{0.0289kg}{\min}\\ R(5) &= \frac{m(5.01) - m(4.99)}{5.01 - 4.99} = -\frac{0.101g}{s}\\ &= -\frac{0.101g}{s} \cdot \frac{1kg}{1000g} \cdot \frac{60s}{1\min} = -\frac{0.00606kg}{\min}\end{aligned}$$

We could use calculus. The rate of change is the derivative so

$$\frac{dm(t)}{dt} = 4.00t^{-0.2} - 3.00$$
$$\frac{dm(2)}{dt} = 4.00 \cdot 2.0^{-0.2} - 3.00 = \frac{0.482g}{s}$$
$$\frac{dm(2)}{dt} = 4.00 \cdot 5.0^{-0.2} - 3.00 = -\frac{0.101g}{s}$$

So our calculus result matches our graphical technique. (We'd still need to convert to kg/min too).

1.35 An old English children's rhyme states "Little Miss Muffet sat on a tuffet, eating her curds and whey, when along came a spider who sat down beside her..." The spider sat down not because of the curds and whey but because Miss Muffet had a stash of 11 tuffets of dried flies. The volume measure of a tuffet is given by 1 tuffet = 2 pecks = 0.5 Imperial bushel = 36.3687 liters (L). What was Miss Muffet's stash in (a)pecks, (b) Imperial bushels, and (c) liters

$$V = 11 tuffets \cdot \frac{2 peck}{1 tuffet} = 22 peck$$
$$V = 11 tuffets \cdot \frac{\text{0.5 Imperial bushes}}{1 tuffet} = \text{5.5 Imperial bushes}$$
$$V = 11 tuffets \cdot \frac{\text{36.3687 L}}{1 tuffet} = 400.056 L$$

1.37 A typical sugar cube has an edge length of 1 cm. If you had a cubical box that contained a mole of sugar cubes, what would its edge length be?

Each sugar cube has a volume of 1 cubic cm. We can find the total volume of the box that will hold a mole of cubes.

$$V = 6.02 \times 10^{23} cubes \cdot \frac{1 cm^3}{cube} = 6.02 \times 10^{23} cm^3$$

Now we can find the edge length of the box.

$$L^3 = 6.02 \times 10^{23} cm^3$$
$$L = 8.44 \times 10^7 cm$$

1.44 What mass of water fell on the town in problem 9

Let's calculate the volume of water in MKS units and then find mass.

$$d = 2in \cdot \frac{2.54cm}{1in} \cdot \frac{1m}{100cm} = 5.08 \times 10^{-2} m$$

$$A = 26\,km^2 \cdot \left(\frac{1000m}{1\,km}\right)^2 = 2.6 \times 10^7\,m$$

$$V = A \cdot d = 2.6 \times 10^7 m \cdot 5.08 \times 10^{-2} m = 1.32 \times 10^6 m^3$$

$$m = V \cdot \rho = 1.32 \times 10^6 m^3 \cdot \frac{1000\,kg}{1\,m^3} = 1.32 \times 10^9\,kg$$